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Contents

Tax Progressivity and Social Welfare with a Continuum of Inequality Views[*]

Marko Ledić[♣], Ivica Rubil[♦], and Ivica Urban[♠]

Abstract

If public policies should aim at promoting social welfare, then tax progressivity/regressivity should be considered in terms of its impact on social welfare, rather than as an end in itself. Whether a tax is progressive or regressive and how much it affects social welfare depends on how a neutral tax, a tax neither progressive nor regressive, is defined. This, in turn, depends on the inequality view taken, that is, on what kind of transformation of an income distribution is considered not to change the level of inequality. Kakwani and Son (Journal of Economic Inequality, 2021) developed a social welfare-based framework, which enables one to decompose the total welfare loss associated with a tax into elements of which one is the welfare impact of tax progressivity/regressivity. While Kakwani and Son consider only the inequality views known as relative and absolute inequality, we provide a generalisation of the framework to accommodate all intermediate inequality views in the continuum between the two polar views. While the total welfare loss does not depend on inequality view, its composition does: for a progressive (regressive) tax, moving closer to the relative view reduces (increases) the importance of progressivity (regressivity) for the total welfare impact. Thus, the perception of the composition of a given tax-induced welfare loss varies with the inequality view taken. We apply the generalised framework to assess the impact on social welfare of the Croatian tax system, showing that it matters for such an assessment which inequality view is taken.

Keywords: progressivity; regressivity; neutrality; relative inequality; absolute inequality; intermediate inequality

JEL classification: H23; H24; I31; I38

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Progresivnost poreza i društveno blagostanje s kontinuumom pogleda na nejednakosti

Sažetak

Ako je svrha javnih politika podizanje društvenoga blagostanja, onda progresivnost/regresivnost poreza treba razmatrati kroz njezin utjecaj na društveno blagostanje, a ne kao cilj po sebi. Je li neki porez progresivan ili regresivan, te koliko utječe na društveno blagostanje, to ovisi o definiciji neutralnoga poreza, naime poreza koji nije ni progresivan ni regresivan. Ta definicija, pak, ovisi o pogledu na nejednakosti, odnosno o tome za kakvu se transformaciju distribucije dohotka smatra da ne mijenja razinu nejednakosti. Kakwani i Son (Journal of Economic Inequality, 2021) razvili su okvir utemeljen na društvenom blagostanju, a koji omogućuje da se ukupni gubitak blagostanja, do kojega dolazi uslijed oporezivanja, dekomponira na elemente među kojima je utjecaj progresivnosti/regresivnosti na društveno blagostanje. Dok Kakwani i Son razmatraju samo poglede na nejednakosti znane kao relativne i apsolutne nejednakosti, mi poopćujemo njihov okvira kako bi uključivao i sve intermedijarne poglede na nejednakosti u kontinuumu između dvaju polarnih pogleda. Ukupni gubitak blagostanja ne ovisi o pogledu na nejednakosti, ali njegova kompozicija ovisi: kod progresivnoga (regresivnoga) poreza, približavanje relativnom pogledu povećava (smanjuje) važnost progresivnosti (regresivnosti) za ukupni utjecaj na blagostanje. Dakle, percipirana kompozicija porezom uzrokovanoga gubitka blagostanja varira s pogledom na nejednakosti. Poopćeni okvir primjenjujemo kako bismo procijenili utjecaj hrvatskoga poreznog sustava na društveno blagostanje, te pokazujemo da je kod takvih procjena važno koji pogled na nejednakosti zauzimamo.

Ključne riječi: progresivnost; regresivnost; neutralnost; relativne nejednakosti; apsolutne nejednakosti; intermedijarne nejednakosti

JEL klasifikacija: H23; H24; I31; I38

1 Introduction

Although taxes exist primarily as a means for governments to finance public expenditures, the political attractiveness of a tax is hardly judged solely on the base of its revenue-raising potential. In particular, the public tends to demand of taxes to be equitable[2], and thus progressivity has traditionally been defended as a key element of tax design that helps the society to achieve a more equitable distribution of income and, consequently, of living standard, by means of fiscal redistribution. In line with this attitude on the part of the public, economists have conceptualised the value of equity itself as deriving from its contribution to social welfare.

The economics literature has long established the relationship between tax progressivity and income inequality (e.g., Jakobsson 1976; Kakwani 1977; Lambert 1993).[3] Likewise, it has long established the relationship between income inequality and social welfare function as an objective function embodying in a formally explicit way both the efficiency and equity considerations (Kolm 1969; Atkinson 1970; Sen 1974).[4] This chain of relationships suggests that tax progressivity is related to social welfare, but unlike the progressivity-inequality and inequality-welfare relationships, the progressivity-welfare relationship has not been explicitly explored until very recently. The literature on optimal nonlinear income taxation derives a set of marginal tax rates that maximise a (typically utilitarian) social welfare function, subject to a resource constraint and incentive compatibility constraints (e.g., Mirrlees 1971; Toumala 1984; Diamond 1998; Saez 2001).[5] In so far as the profile of optimal marginal tax rates across the distribution of pre-tax income determines the profile of average tax rates, which in turn renders the tax progressive, regressive, or neutral, this literature could be interpreted as relating tax progressivity with social welfare. However, in this way, the two are related only implicitly, as a by-product, and not directly enough to make the relationship clear.

In a recent paper, Kakwani and Son (2021) (hereafter: K&S) have developed an analytical framework based on social welfare function, explicating the impact of taxation, and tax progressivity in particular, on social welfare. As a general approach, they consider the social welfare impact of taxation, expressed as the difference in the level of social welfare after and before a given tax.[6] The difference, which cannot be positive, but zero at best – i.e., taxation cannot be welfare-improving, only welfare-neutral at best – is decomposed into three effects. First, the effect of a counterfactual neutral (i.e., neither progressive nor regressive) tax yielding the same revenue as the actual tax. This effect, called the "neutral-tax effect" is always

[2] See Saez (2021) for an illuminating account of this.
[3] For a review, see Lambert (2001).
[4] For a review, see Adler (2019).
[5] For a review, see Toumala (2016).
[6] Kakwani et al. (2021) have developed a similar social welfare-based framework dealing with social benefits (i.e., transfers), rather than taxes.

welfare-reducing. Second, the effect stemming from the tax's progressivity or regressivity (i.e., non-neutrality), called the "progressivity effect", which is welfare-increasing if the tax is progressive, and welfare-reducing if the tax is regressive. And third, the effect arising from the tax's horizontal inequity, manifested as reranking between tax units, called the "horizontal effect". If there is reranking, this effect is welfare-reducing, while otherwise it has no impact on welfare.

K&S derive the decomposition for a number of specific social welfare functions: namely, the S-Gini (or generalised Gini) (Kakwani 1980; Donaldson & Weymark, 1980, 1983; Yitzhaki, 1983), the Atkinson (Atkinson 1970), the Kolm (Kolm 1976a, 1976b) and the Bonferroni (Son 2011) social welfare functions.[7] In each case, the progressivity effect is interpreted as a measure of tax progressivity with an explicit link to the underlying social welfare function. Of the functions used by K&S, only those from the S-Gini class can be the base for inequality indices that reflects two alternative inequality views, the absolute and the relative, introduced by Kolm (1976a, 1976b). According to the relative inequality view, inequality remains unchanged if all incomes are changed by the same relative amount; that is, if all incomes are multiplied by the same number. According to the absolute view, inequality remains unchanged if the same amount is added to, or subtracted from, all individuals' incomes. K&S exploit this and derive two versions of the decomposition, one based on the relative, and the other on the absolute inequality view. The key difference between them is in the definition of a neutral tax, i.e., the one that is neither progressive nor regressive. While in the relative version a neutral tax is a proportional tax (Kakwani 1977) – everyone pays the same proportion of the pre-tax income – in the absolute version a neutral tax is a uniform tax – everyone pays the same absolute amount of tax. The progressivity effect is taken as to be a social welfare-based measure of tax progressivity/regressivity, measuring the social welfare impact of progressivity or regressivity as departures from proportionality or uniformity, respectively. In the relative version, the progressivity effect turns out to be equivalent to the standard Kakwani index progressivity (Kakwani 1977).

The absolute and the relative inequality views are, however, just two polar or extreme views, while there are other views possible, usually referred to as "intermediate" inequality views (Bossert and Pfingsten 1990; del Río and Ruiz-Castillo 2000, 2001; Ebert 2004; Yoshida 2005; Bosmans et al. 2014). Intermediate views can be seen as certain combinations of the absolute and the relative views. In this paper, we contribute by generalising the K&S framework to obtain a framework that nests their absolute and relative versions of the decomposition as polar special cases, along with the continuum of versions based on intermediate inequality views within the continuum of views ranging from the absolute to the relative. To do that, we adopt the approach to intermediate inequality of del Río and Ruiz-Castillo (2000, 2001) and Bosmans et al. (2014),

[7] Greselin et al. (2020) applied the K&S general framework to derive the decomposition for the SWF underlying the Zenga inequality index.

termed “(x, π)-inequality” by the former and "α-inequality" by the latter authors. According to this approach, inequality does not change when all incomes are changed by the same relative amount *and* by the same absolute amount, in proportions α and $1 - \alpha$, respectively, with $0 \leq \alpha \leq 1$. The absolute view and the relative view are obtained as the special cases corresponding to $\alpha = 0$ and $\alpha = 1$, respectively. Being based on the α-inequality approach, our generalisation builds upon Urban's (2019) generalisation of the Kakwani progressivity index to accommodate inequality views other than the relative, on which the standard Kakwani index is based (Kakwani 1977). In fact, just as K&S's relative version provides a social welfare-based interpretation of the standard Kakwani index, our generalised version provides such interpretation to Urban's (2019) index of α-progressivity.

We show that the generalised decomposition of tax-induced welfare change can be expressed as a weighted average of K&S's absolute and relative versions, with α and $1 - \alpha$ as the respective weights. We also show that, while the total welfare impact of a tax does not depend on inequality view, its composition does. In particular, for a progressive tax, moving closer to the relative view (i.e., increasing α) reduces the importance of progressivity for the total welfare impact: the welfare-increasing progressivity effect becomes smaller relative to the welfare-reducing neutral-tax effect. Conversely, for a regressive tax, getting closer to the relative view increases the importance of regressivity for the total welfare impact: the welfare-reducing effect of regressivity becomes larger relative to the welfare-reducing neutral-tax effect.

In an illustrating empirical application, we use the generalised framework to assess the welfare impact of taxation in Croatia in 2017, considering personal income tax, social insurance contributions, and indirect taxes (VAT and excises). The results suggest that it matters substantially which inequality view is taken while assessing the welfare impact of taxation, and the framework we propose allows for a whole continuum of inequality views to be considered and compared.

The rest of the paper is organised in this way: section 2 presents K&S's social welfare framework; in section 3, we provide our generalisation; section 4 is the empirical application; section 5 summarises the paper and concludes.

2 Taxation and social welfare: a framework

In this section, we present the framework of K&S for the analysis of the impact of taxation on social welfare. Section 2.1 introduces notation and the social welfare function used. K&S's decomposition of the welfare impact of taxation is derived in section 2.2.[8]

[8] Our notation does not fully correspond to K&S’s notation.

2.1 Taxation and the S-Gini social welfare function

Denote pre-tax and post-tax incomes by x and $y = y(x)$, respectively. The two differ by the amount of tax, $t(x)$: $t(x) = x - y(x)$. Denote the means of x, y, and t by μ_x, μ_y, and μ_t, respectively.[9] Let the density functions of x and y be, respectively, $f(x)$ and $f^*(y)$, with the corresponding cumulative distribution functions $F(x)$ and $F^*(y)$.

K&S's framework accommodates a number of different social welfare functions (SWF's), including the S-Gini class (Kakwani 1980; Donaldson & Weymark, 1980, 1983; Yitzhaki, 1983), the Atkinson class (Atkinson 1970), the Kolm SWF (Kolm 1976a, 1976b), and the Bonferroni SWF (Son 2011). Here we consider the S-Gini class only, and for the following reason. Unlike the other SWF's considered by K&S, the S-Gini class can be the base for both relative and absolute inequality and concentration indices, that is, it accommodates both the relative and absolute inequality views. K&S exploit this feature and derive their framework in two versions, one relying on the relative, the other on the absolute inequality view. This is crucial for us, as in this paper we aim to provide a generalised framework which accommodates not only the two polar inequality views, but intermediate views as well.

The S-Gini class of SWF's is a class of rank-dependent SWF's, where each person's income is weighted by a weight which depends on her relative rank in the income distribution. Assuming that all pre-tax incomes are non-negative, the pre-tax social welfare is

$$W_x := \int_0^\infty x\, \omega(F(x), \rho) f(x) \mathrm{d}x, \tag{1}$$

where $\omega(F(\cdot), \rho)$ is the S-Gini social welfare weight. The latter takes the form

$$\omega(F(x), \rho) := \rho\big(1 - F(x)\big)^{\rho - 1}, \tag{2}$$

where ρ is an ethical parameter. For any $\rho > 1$, the weight is decreasing with income, and so is the marginal social welfare associated with marginal increase of income of those with income equal to x. The larger the ρ, the faster the weight decreases with income, and the larger the difference in the weight between two persons with different incomes (i.e., different ranks in the distribution).[10,11] Moreover, the weights integrate to one: $\int_0^\infty \omega(F(x), \rho)\, f(x) \mathrm{d}x = 1$. Fixing ρ at a specific value amounts to choosing a specific SWF from

[9] We slightly abuse notation: for simplicity, throughout the paper we use lowercase letters for both variables and their specific values. For example, x denotes pre-tax income both as a variable and its specific value. In addition, statistics computed on a vector of values of a variable will be subscripted by the appropriate lowercase letter: for example, the mean pre-tax income is denoted by μ_x.

[10] In other words, the larger the ρ, the steeper the curve depicting $\omega(F(z), \rho)$ as a function of $F(z)$. For a detailed discussion, see Araar and Duclos (2006).

[11] Note that in (1), by using the notation W_x, we do not indicate that the social welfare depends on ρ. We do so for notational simplicity only. We follow this convention throughout the paper, so that any expression depending on ρ is written for a given value of this parameter.

the S-Gini class. For example, $\rho = 2$ yields the Sen SWF (Sen 1974), the one that underlies the standard Gini coefficient. In parallel with (1), the post-tax social welfare is given by:

$$W_y \coloneqq \int_0^\infty y\,\omega(F^*(y),\rho) f^*(y)\mathrm{d}y. \tag{3}$$

The order obtained by sorting the population from the lowest to the highest post-tax income need not be identical to the order obtained when the population is sorted in ascending order of pre-tax income, as the tax may lead to some reranking of persons. To recognise this is critical for the definition of a different notion of social welfare than that underlying (2) and (3), which we require for the analysis to follow. Imagine that the population is sorted by post-tax income, but that in calculating the post-tax social welfare the ω-weights are ascribed to individuals as if they were sorted in ascending order of pre-tax income. In other words, imagine that the social welfare associated with post-tax incomes is obtained using the pre-tax instead of the post-tax weighting scheme:

$$\Psi_y \coloneqq \int_0^\infty y(x)\,\omega(F(x),\rho) f(x)\mathrm{d}x. \tag{4}$$

K&S call Ψ_y the "pseudo social welfare" and prove that $\Psi_y \geq W_y$ (K&S, Appendix, Theorem A.2), meaning that reranking always leads to a welfare loss. For later use[12], let us also define

$$\Psi_t \coloneqq W_x - \Psi_y = \int_0^\infty t(x)\,\omega(F(t),\rho) f(x)\mathrm{d}x. \tag{5}$$

With the above notation, for pre- and post-tax incomes, the S-Gini social welfare and the corresponding S-Gini coefficients are related as follows:

$$W_x = \mu_x(1 - G_x), \quad \text{i.e.} \quad G_x = 1 - \frac{W_x}{\mu_x}, \tag{6a}$$

$$W_y = \mu_y\left(1 - G_y\right), \quad \text{i.e.} \quad G_y = 1 - \frac{W_y}{\mu_y}, \tag{6b}$$

where G_x and G_y stand for the S-Gini coefficients of pre- and post-tax income, respectively. In addition,

$$\Psi_y = \mu_y\left(1 - D_y\right), \quad \text{i.e.} \quad D_y = 1 - \frac{\Psi_y}{\mu_y}, \tag{7a}$$

$$\Psi_t = \mu_t(1 - D_t), \quad \text{i.e.} \quad D_t = 1 - \frac{\Psi_t}{\mu_t}, \tag{7b}$$

[12] It is used in equation (7b), to appear shortly.

where D_y and D_t are the S-concentration coefficients of y and t, respectively, when the population is sorted by pre-tax income x.[13] As we said above, K&S prove that that $\Psi_y \geq W_y$, implying that $D_y \leq G_y$ always holds as well.

What enables the S-Gini class to be the base of both relative and absolute inequality indices is the fact that a SWF from this class is both relatively and absolutely homogeneous of degree one. Relative homogeneity of degree one means that if all incomes are multiplied by the same factor λ, the level of social welfare is also scaled by the same factor: $W_{\lambda x} = \lambda W_x$, $\lambda > 0$ (and analogously for W_y, Ψ_y, and Ψ_t). Absolute homogeneity of degree one means that if all incomes are added or subtracted the same amount υ, the level of social welfare changes by that amount: $W_{x+\upsilon} = W_x + \upsilon$ (and analogously for W_y, Ψ_y, and Ψ_t).

2.2 The welfare impact of taxation and its decomposition

In the transition from pre-tax to post-tax income the S-Gini social welfare is changed by the amount $W_y - W_x$. K&S propose the following decomposition of the welfare change, which holds for any S-Gini parameter $\rho > 1$:

$$W_y - W_x = \underbrace{-\mu_t(1 - G_x)}_{\text{neutral-tax effect}} + \underbrace{\mu_t(D_t - G_x)}_{\text{progressivity effect}} + \underbrace{\left(W_y - \Psi_y\right)}_{\text{horizontal effect}}, \tag{8}$$

where $\tau \equiv \mu_t/\mu_x$ is the average tax rate. The first term on the right-hand side of (8), $-\mu_t(1 - G_x)$, called the neutral-tax effect, is the welfare loss that would result from imposing a counterfactual proportional equal-yield tax, $t_p = \tau x$, instead of the actual tax t. Levying t_p would collect the same tax revenue as t, by taking from everyone the same proportion of the pre-tax income, and causing no changes in relative inequality level and no reranking. The second term, $\mu_t(D_t - G_x)$, called the progressivity effect, measures the welfare change due to the departure of the actual tax from proportionality. More precisely, it measures the increase (decrease) of welfare caused by the tax system's progressivity (regressivity). Finally, the horizontal effect, $W_y - \Psi_y$, represents the loss of welfare due to reranking of income units caused by the tax, a manifestation of horizontal inequity. Reranking does not cause a welfare loss *per se*, as the social welfare function does not embody aversion to horizontal inequity. Rather, reranking is welfare-reducing only insofar as it offsets the welfare-increasing effect of progressivity. Added up together, the progressivity and horizontal effects make up what may be called the revenue-neutral inequality effect, measuring the welfare change on account of the tax-induced change in relative inequality, keeping the tax revenue unchanged.

[13] Since the sorting variable for all concentration coefficients that we use is x, for simplicity we do not indicate that in notation.

Decomposition (8) reflects the relative inequality view, which holds that inequality remains unchanged if taxation changes the incomes of all persons in equal proportion. A proportional tax, which takes the same proportion of everyone's pre-tax income, represents such a change. In contrast, the absolute inequality view holds that inequality is unchanged if all incomes are changed by an equal absolute amount. Besides decomposition (8), K&S also derive the following decomposition of the welfare change, which reflects the absolute inequality view:

$$W_y - W_x = -\mu_t + \mu_t D_t + \left(W_y - \Psi_y\right). \tag{9}$$

The switch from the relative to the absolute inequality view does not affect the size of the welfare impact being decomposed; what changes only is how it is decomposed. The neutral-tax effect, $-\mu_t$, now shows the welfare reduction due to a counterfactual uniform equal-yield tax, $t_u = \tau\mu_x = \mu_t$. This tax yields the same tax revenue as the actual tax t, while keeping absolute inequality unchanged. Contrast this with the previously introduced counterfactual proportional equal-yield tax $t_p = \tau x$, which yields the same tax revenue, while keeping relative inequality unchanged.

Dividing the decompositions (8) and (9) by μ_t, one obtains the decomposition of welfare change per unit of tax revenue for the relative and absolute inequality views, respectively, as

$$\frac{W_y - W_x}{\mu_t} = -(1 - G_x) + (D_t - G_x) + \frac{W_y - \Psi_y}{\mu_t}, \tag{10}$$

$$\frac{W_y - W_x}{\mu_t} = -1 + D_t + \frac{W_y - \Psi_y}{\mu_t}, \tag{11}$$

where $D_t - G_x$ and D_t represent the Kakwani index of progressivity (Kakwani, 1977) for the relative and the absolute inequality views, respectively. In each decomposition, the Kakwani index has a clear interpretation: it represents the welfare gain (loss) per unit of tax revenue due to tax progressivity (regressivity). Expressing the welfare change per unit of tax revenue, as in (10) and (11), is useful as it makes possible comparisons between the welfare impacts of taxes differing in size (i.e., differing in the average tax rate). Note that the horizontal effect, reflecting the welfare loss due to reranking, is same for both inequality views, since the phenomenon of reranking is independent of how the standard of neutrality is defined.[14]

3 A generalisation to a continuum of inequality views

In this section, we generalise the K&S decomposition of the social welfare impact of taxation by considering not only the relative and absolute inequality views (equations (10) and (11), respectively), but the whole

[14] In K&S, the horizontal effects in the relative and absolute decompositions are not the same, but only because they use the equation (9).

continuum of intermediate inequality views in between the two polar views. Again, the following presentation holds, without any modification, for any $\rho > 1$.

3.1 The Kakwani index with a continuum of inequality views

At the end of section 2.2, we mentioned the relative and absolute Kakwani indices. The relative one is probably the most popular and widely used scalar measure of tax progressivity. In contrast, its absolute counterpart is rarely used in applied work, arguably because the standard analysis of inequality in general – and of income redistribution in particular – implicitly assumes the relative inequality view. Despite the rich literature on income inequality and tax progressivity for alternative inequality views, they are still seldom present in empirical research. In the remainder of this section, we show how the standard Kakwani index can be generalized to accommodate various inequality views.

The relative Kakwani index,

$$K_{\text{rel}} := D_t - G_x, \tag{12}$$

measures the deviation of the actual tax from proportionality. To illustrate this, imagine a tax $t_p = \tau x$, which provides the same amount of tax revenue as the actual tax t, but is proportional to pre-tax income. Because the Gini and concentration indices are relative inequality measures (i.e., invariate to equi-proportional changes in the income distribution), the concentration index of the hypothetical proportional tax is equal to the Gini index of pre-tax income: $D_{t_p} = G_x$. Therefore, the equation (12) becomes

$$K_{\text{rel}} = D_t - D_{t_p}, \tag{13}$$

which clarifies the interpretation of K_{rel}: it measures the extent of deviation from proportionality. A tax t is progressive (regressive) (neutral) if $D_t > D_{t_p}$ ($D_t < D_{t_p}$) ($D_t = D_{t_p}$), that is, if it is more (less) (equally) concentrated along the distribution of pre-tax income than the proportional tax t_p.

Thus, it is the proportional tax that is used here as the reference to determine whether a tax is progressive or regressive. Put differently, proportionality is taken as the standard of inequality neutrality. This is but one possibility, however, reflecting one polar inequality view, namely the relative view. Alternative possibilities, reflecting alternative inequality views, are available as well. Switching from one inequality view to another boils down to choosing a different reference tax, one that is neutral according to a different standard of neutrality. Choose, for instance, a uniform (lump-sum) tax yielding the same revenue as the actual tax, $t_u = \mu_t$, as the reference neutral tax; that is, take "uniformity" as the standard of neutrality. This choice brings us to the other polar inequality view, the absolute view. As for the relative inequality view, where the Kakwani index is the difference between the concentration indices of the actual and the proportional reference tax, for the absolute view the Kakwani index is obtained as the difference between the concentration index of the actual and the uniform reference tax. Thus, the Kakwani index for the absolute view is

$$K_{\text{abs}} := D_t - D_{t_u} = D_t, \tag{14}$$

where the second equality is due to $D_{t_u} = 0$.

The range of possibilities is not exhausted by the two polar views, however, as there is a continuum of intermediate inequality views in between the relative and absolute poles. Relying on the concept of intermediate inequality of del Río and Ruiz-Castillo (2000; 2001) and Bosmans et al. (2014), called (x, π)-inequality by the former authors, and α-inequality by the latter, Urban (2019) proposes a generalized version of the Kakwani index, one which incorporates the continuum of inequality views.[15] First, for the actual tax define its α-inequality-neutral equal-yield counterpart:

$$t_\alpha := \alpha\tau x + (1-\alpha)\tau\mu_x = \alpha\tau x + (1-\alpha)\mu_t, \tag{15}$$

where α is the parameter of inequality view. For $\alpha = 1$ and $\alpha = 0$, the above discussed relative and absolute views are obtained, respectively; for other values of α, a range of intermediate inequality views are obtained. t_α is the reference neutral tax corresponding to one in the range of α-inequality views. This reference tax is in accordance with the standard of neutrality obtained by combining proportionality and uniformity in proportions α and $1-\alpha$, respectively. The generalized, α-Kakwani index is given by

$$K_\alpha := D_t - D_{t_\alpha} = D_t - \alpha G_x, \tag{16}$$

where the second equality is due to the fact $D_{t_\alpha} = \alpha G_x$.[16] Note that this equality means that obtaining D_{t_α} does not require obtaining t_α beforehand; it suffices to choose α and to know G_x.[17] The Kakwani indices corresponding to the relative and absolute inequality views are special cases of K_α: $K_{\text{rel}} = K_1$ and $K_{\text{abs}} = K_0$, respectively.

A given tax may be progressive for some values of α, and regressive for other values: that is, for given D_t and G_x, the nature of a tax's departure from neutrality may vary with α. Denote by $\tilde{\alpha}$ the value of α for which the tax is α-neutral, that is, for which the generalised Kakwani index K_α is zero. Thus, $\tilde{\alpha}$ is defined by $D_t - \tilde{\alpha}G_x = 0$, implying $\tilde{\alpha} = D_t/G_x$. For all $\alpha > \tilde{\alpha}$, the tax is regressive ($K_\alpha < 0$), while for all $\alpha < \tilde{\alpha}$, the taxis progressive ($K_\alpha > 0$). Assuming $G_x > 0$, we can differentiate five cases.

Case 1: $D_t > G_x$. In this case, $\tilde{\alpha} > 1$, and thus the tax is progressive for all $\alpha \in [0,1]$. An example is a tax where the average tax rate at the level of pre-tax income x, $\tau(x) = t(x)/x$ (i.e., the fraction of x taxed away), increases with x. Consequently, the tax liability, $t(x)$, also increases with x.

[15] Urban (2019) refers to Bosmans et al. (2014) as the originators of this approach to intermediate inequality.
[16] For a proof, see section A.1 in Appendix.
[17] This fact was not noted in Urban (2019).

Case 2: $D_t = G_x$. In this case, $\tilde{\alpha} = 1$, and thus the tax is neutral for $\alpha = 1$, and progressive for all $\alpha \in [0,1)$. That is, the tax is neutral according to the relative view, and progressive for all other views. An example is a proportional tax where $\tau(x)$ does not vary with x, but rather equals the overall average tax rate: $\tau(x) = \tau = \mu_t/\mu_x$. Consequently, $t(x)$ increases with x.

Case 3: $0 < D_t < G_x$. In this case, $0 < \tilde{\alpha} < 1$, and thus the tax is progressive for all $\alpha \in [0, D_t/G_x)$, neutral for $\alpha = D_t/G_x$, and regressive for all $\alpha \in (D_t/G_x, 1]$. An example is a tax where $\tau(x)$ decreases with x, while $t(x)$ increases with x.

Case 4: $D_t = 0$. In this case, $\tilde{\alpha} = 0$, and thus the tax is neutral for $\alpha = 0$, and regressive for all $\alpha \in (0,1]$. An example is a uniform tax where $t(x)$ does not vary with x, but rather equals the average tax amount, $t(x) = \mu_t$. Consequently, $\tau(x)$ decreases with x.

Case 5: $D_t < 0$. In this case, $\tilde{\alpha} < 0$, and thus the tax is regressive for all $\alpha \in [0,1]$. An example is a tax where both $t(x)$ and $\tau(x)$ decrease with x.

Not all these cases are equally likely to be part of a real-world tax system. Indeed, a tax exemplary of case 5 is hardly observed in any country. Perhaps the closest to it would be an excise on a good or service whose absolute consumption (i.e., not only its budget share) decreases with pre-tax income. Cases 2 and 4 are not so unlikely to exist, although certainly not in the exact form presented above, but only approximately. As we will see later on, social insurance contributions in Croatia correspond approximately to case 2. Cases 1 and 3 are regularly observed in the real world as personal income tax and the value added tax (or all indirect taxes considered together), respectively.

3.2 Decomposing the welfare impact of taxation in the α-inequality framework

As we have seen in section 2.2, K&S proposed decompositions of the welfare impact of taxation for the relative and absolute inequality views. Here we generalise their decomposition framework by combining it with the above discussed Urban's (2019) generalisation of the Kakwani index to intermediate inequality views. We derive a generalised decomposition, which allows the implementation of a whole range of inequality views. The generalised decomposition nests the decompositions from equations (10) and (11) as special cases.

We start with the concept of counterfactual post-tax income, y_α, obtained by subtracting the equal-yield α-neutral tax, as defined in (15), from pre-fiscal income: $y_\alpha = x - t_\alpha$. According to the α-inequality concept, y_α has same inequality as x. Moreover, since t_α yields the same revenue as t, we have $\mu_{t_\alpha} = \mu_t$, and consequently $\mu_{y_\alpha} = \mu_y$. The pseudo social welfare arising from y_α, is given by

$$\Omega_\alpha := \int_0^\infty y_\alpha(x)\,\omega(F(x),\rho) f(x) \mathrm{d}x = \mu_y \left(1 - D_{y_\alpha}\right), \tag{17}$$

This is the level of welfare achieved without affecting α-inequality, while collecting the same amount of revenue as with the actual tax. Note that unlike other levels of welfare (W_x, W_y) or pseudo welfare (Ψ_y, Ψ_t), this one depends on α.

The welfare impact of taxation per unit of tax collected is decomposed as follows:[18]

$$\Delta W = N(\alpha) + P(\alpha) + H, \tag{18a}$$

where

$$\Delta W := \frac{1}{\mu_t}\left(W_y - W_x\right), \tag{18b}$$

$$N(\alpha) := \frac{1}{\mu_t}(\Omega_\alpha - W_x) = -(1 - \alpha G_x), \tag{18c}$$

$$P(\alpha) := \frac{1}{\mu_t}\left(\Psi_y - \Omega_\alpha\right) = D_t - \alpha G_x, \tag{18d}$$

$$H := \frac{1}{\mu_t}\left(W_y - \Psi_y\right) = \frac{\mu_y}{\mu_t}\left(D_y - G_y\right). \tag{18e}$$

By setting $\alpha = 1$ and $\alpha = 0$, equation (18a) turns into the decomposition of the welfare impact of taxation for the relative and absolute inequality view, respectively, while an α between 0 and 1 yields an intermediate, α-specific decomposition. An α-specific decomposition can be interpreted as a weighted average of the decompositions specific to the relative and absolute inequality views, with the weights α and $(1 - \alpha)$, respectively. This can be verified by multiplying (10) by α and (11) by $(1 - \alpha)$, and adding them up to obtain (18a).

The first term on the right-hand side of (18a), the α-neutral tax effect $N(\alpha)$, is the welfare impact of the α-neutral equal-yield tax t_α. Precisely, it is the welfare loss per dollar of the tax: if the tax were α-neutral, each dollar the tax raised would reduce social welfare by $N(\alpha)$ dollars. The second term, the α-progressivity effect $P(\alpha)$, represents the welfare impact of the departure of the tax from α-neutrality; it is positive (negative) (zero) or welfare-increasing (welfare-reducing) (welfare-neutral) if the tax is α-progressive (α-regressive) (α-neutral). This effect is equal to the generalised Kakwani index of Urban (2019), given by equation (16) above. Hence, the five cases discussed in section 3.1 apply here as well. The third term, the horizontal effect H, measures the welfare impact of reranking caused by the tax, which may be either negative (welfare-reducing) or zero (welfare-neutral), but may not be positive (welfare-increasing).[19]

Note that when the tax is α-neutral, then not only $P(\alpha) = 0$ (by definition of α-neutrality), but also $H = 0$, as an α-neutral tax cannot cause reranking.[20] Intuitively, it is obvious that neither a relatively neutral

[18] For a proof, see section A.2 in Appendix.
[19] K&S prove that that $\Psi_y \geq W_y$, implying that $D_y \leq G_y$. Thus, $H > 0$ is never true.
[20] For a proof, see section A.3 in Appendix.

nor an absolutely neutral tax can cause reranking. If reranking cannot occur in these polar cases, then it cannot occur in an intermediate case either, which is a combination of the polar cases.

Moreover, note that the welfare loss due to $N(\alpha)$ and H (if there is reranking) cannot be outweighed by a sufficiently large positive $P(\alpha)$; at best, $P(\alpha)$ can fully offset the welfare reduction arising from the other two effects. Thus, the total welfare impact of a tax cannot be positive; it can be zero at best.[21] The conditions that must be satisfied for the total welfare impact to be zero are: (i) the only taxpayer is the person with the highest pre-tax income ($D_t = 1$), and (ii) the tax does not change this person's rank ($H = 0$).[22] The condition (ii) implies that the taxpayer's tax liability, which is the aggregate tax collected, must be equal to the difference between the taxpayer's pre-tax income and the pre-tax income of the second richest person in the pre-tax distribution. While $H = 0$, or at least $H \approx 0$, is typical in practice, $D_t = 1$ is practically impossible. Thus, in all relevant cases, the total welfare impact is a welfare loss, $\Delta W < 0$.

3.3 How the decomposition changes with inequality view

The total welfare loss and the horizontal effect are the same as in decompositions (10) and (11) since they are defined with no reference to the standard of neutrality: as indicated by expressions (18b) and (18e), neither depends on α. In contrast, the neutral-tax and progressivity effects are inequality view-specific and thus depend on α, as can be seen from expressions (18c) and (18d). How do changes in the parameter α affect the decomposition (18a)? Since ΔW and H are invariant to changes in α, when α changes the other two effects, $N(\alpha)$ and $P(\alpha)$, must change by the same magnitude, but in opposite directions: $\partial N(\alpha)/\partial\alpha = G_x > 0$, $\partial P(\alpha)/\partial\alpha = -G_x < 0$. What the changes in these two effects mean and how should they be interpreted?

Since $N(\alpha) < 0$, an increase in α reduces the absolute value of $N(\alpha)$, which means that the welfare loss of the equal-yield α-neutral tax is monotonically decreasing as one moves from the absolute inequality view towards the relative. To understand why, consider how an α-neutral tax, amounting to one dollar on average, would affect welfare per dollar of tax in the two polar inequality views – the absolute and the relative. By definition, $\Delta W = N(\alpha)$ for an α-neutral tax. Consider first the absolute view. Raising one dollar on average in an absolutely neutral way requires taking one dollar from everyone, which increases the Gini coefficient. Thus, the post-tax social welfare, $W_y = \mu_y(1 - G_y)$ is lower than the pre-tax welfare, $W_x = \mu_x(1 - G_x)$, due to both $\mu_y < \mu_x$ (as $\mu_t = \mu_y - \mu_x > 0$) and $G_y > G_x$. Now consider the relative view. Raising one dollar on average in a relatively neutral way requires taking the same proportion of everyone's pre-tax income, under the constraint that the average tax liability equals one dollar. Since this tax takes the same

[21] For a proof, see section A.4 in Appendix.
[22] For a proof, see section A.5 in Appendix.

proportion of everyone's pre-tax income, the Gini coefficient remains unchanged, and the post-tax welfare is lower than the pre-tax welfare only due to $\mu_y < \mu_x$. Since the average tax liability is the same in both cases, a larger loss is caused by the absolutely neutral tax (for which $G_y > G_x$) than by the relatively neutral tax (for which $G_y = G_x$).

To understand why $P(\alpha)$ decreases with α, note that the condition for α-progressivity (α-regressivity), $P(\alpha) = D_t - \alpha G_x > 0$ ($P(\alpha) = D_t - \alpha G_x < 0$) is more difficult (easy) to satisfy the larger the α (all else equal). This is best seen by comparing the polar inequality views: absolute progressivity is less demanding than relative progressivity. This is because for absolute progressivity it suffices that tax is concentrated among the relatively rich ($D_t > 0$), while relative progressivity in addition requires tax to be more concentrated than pre-tax income ($D_t - G_x > 0$). Therefore, absolute progressivity (regressivity) is necessary (sufficient), but not sufficient (necessary), for relative progressivity (regressivity). In general, if a tax is progressive (regressive) for $\alpha = \tilde{\alpha}$, then it is progressive (regressive) for any smaller (larger) α too (Urban 2019).

Whether the absolute value of $P(\alpha)$ increases or decreases upon an increase in α, depends on the sign of $P(\alpha)$. If $P(\alpha) > 0$ ($P(\alpha) < 0$), i.e., if the tax is α-progressive (α-regressive), an increase in α reduces (increases) its absolute value, meaning that the welfare-increasing (welfare-reducing) impact of α-progressivity (α-regressivity) becomes smaller (larger).

It should be stressed that claims such as "By increasing α, $P(\alpha)$ decreases, meaning that the tax becomes *less progressive* (or *more regressive*)" are senseless. The reason is that to change α means to change the inequality view, and thus the standard of neutrality, and in turn the definition of progressivity. And here we are not referring only to situations such as case 3 in section 3.1 ($0 < D_t < G_x$), where the tax is progressive for α's up to some value, and regressive for those beyond it (an example is the value added tax). Rather, comparing the extent of tax progressivity for different α is senseless even if the tax in question is progressive or regressive for all α. Comparisons of the extent of progressivity make sense only for a given α, i.e., for a given standard of neutrality implied by a specific inequality view. Thus, a claim that makes sense would be: "Tax A is more α-progressive than tax B."

While ΔW remains unchanged as α varies, its composition does change. Precisely, the relative sizes of $P(\alpha)$ and $N(\alpha)$ vary with α in a certain manner. To see that, it is useful to divide both sides of equation (18a) by the absolute value of $N(\alpha)$:

$$\underbrace{\frac{\Delta W}{|N(\alpha)|}}_{:=\,\delta(\alpha)} = -1 + \underbrace{\frac{P(\alpha)}{|N(\alpha)|}}_{:=\pi(\alpha)} + \underbrace{\frac{H}{|N(\alpha)|}}_{:=\,\eta(\alpha)}. \tag{19}$$

The term on the left-hand side, $\delta(\alpha)$, is the actual welfare loss expressed relative to the welfare loss that would obtain if the tax were α-neutral. When the actual tax is α-neutral, $\delta(\alpha)$ equals –1. When the actual tax is not α-neutral, $\delta(\alpha)$ is larger or smaller than –1, depending on the signs of $P(\alpha)$ and H. Recall that $H = 0$ if there is no reranking, and $H < 0$ if there is reranking, so that $\eta(\alpha) \leq 0$. Thus, reranking increases the actual welfare loss above the level that would obtain if the tax were α-neutral. $P(\alpha)$, and consequently $\pi(\alpha)$, is positive (negative) (zero) if the tax is α-progressive (α-regressive) (α-neutral). Therefore, an α-progressive (α-regressive) tax reduces (increases) the actual welfare loss relative to the loss that would prevail if the tax were α-neutral. A positive $\pi(\alpha)$ can be interpreted as measuring the fraction of the welfare loss on account of $N(\alpha)$ that is offset by the welfare gain arising from α-progressivity. Likewise, one can interpret a negative $\pi(\alpha)$ as measuring how much an α-regressive tax contributes to the welfare loss on top of $N(\alpha)$, expressed as a share of $N(\alpha)$.

$\delta(\alpha)$, $\pi(\alpha)$, and $\eta(\alpha)$ all depend on inequality view, and thus change with α. As can be seen from (19), the change in $\delta(\alpha)$ upon an increase in α is the sum of changes in $\pi(\alpha)$ and $\eta(\alpha)$. $\pi(\alpha)$ is decreasing in α, except in a practically irrelevant case where there is only one taxpayer ($D_t = 1$), the person with the highest pre-tax income.[23] This, however, does not mean that $P(\alpha)$ relative to $N(\alpha)$ is monotonically falling as one moves from the absolute towards the relative inequality view. For the values of α for which the tax is progressive (i.e., $\alpha \in [0, \tilde{\alpha})$, where $\tilde{\alpha} := D_t/G_x$), increasing α reduces the relative size of $P(\alpha)$. On the contrary, for the value of α for which the tax is neutral (i.e., $\alpha = \tilde{\alpha}$) and for those for which the tax is regressive (i.e., $\alpha \in (\tilde{\alpha}, 1]$) higher values, increasing α increases the relative size of $P(\alpha)$.

$\eta(\alpha)$ is decreasing in α too, except if there is no reranking (i.e., except if $H = 0$).[24] When $H \neq 0$, the fall in $\eta(\alpha)$ when α increases is proportional to H. Real-world taxes typically cause relatively little reranking once we take account of differences in needs among households through adjusting pre-tax incomes using a household equivalent scale. Thus, typically, $\delta(\alpha)$ will be predominantly determined by $\pi(\alpha)$, rather than $\eta(\alpha)$.

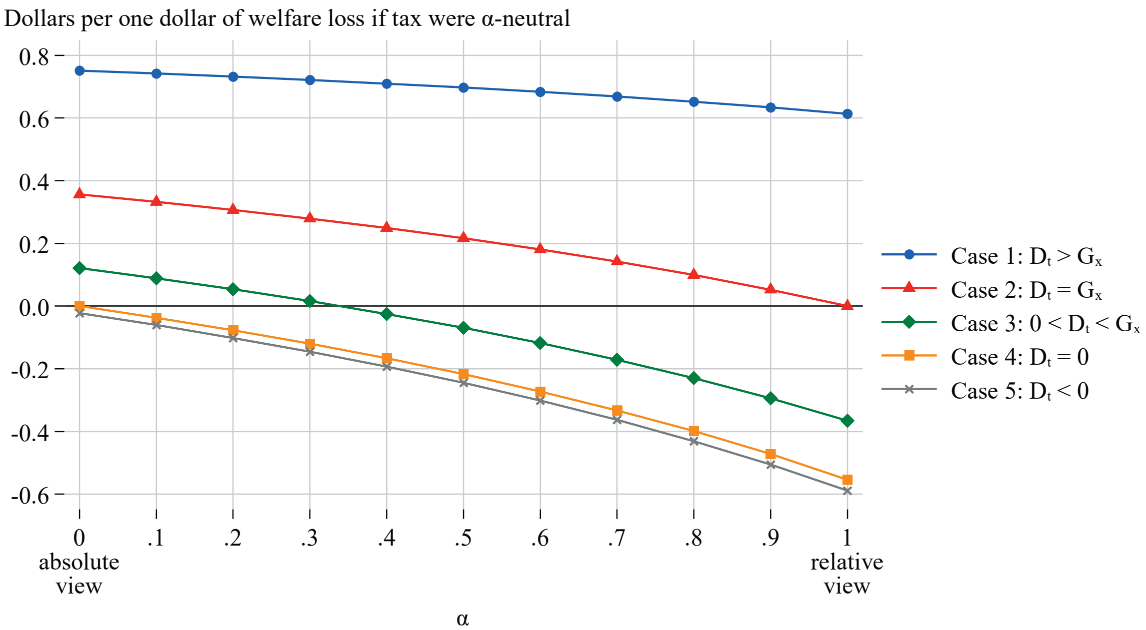
Figure 1 plots $\pi(\alpha)$ against α for five hypothetical taxes, corresponding to the five cases discussed in section 3.1. The figures are specific to the S-Gini parameter $\rho = 2$, associated with the Sen SWF (Sen 1974) and the standard Gini and concentration coefficients. In each case, there is very little or no reranking ($H \approx 0$ or $H = 0$, respectively).[25] In cases 1 and 2, showing taxes which are progressive for all α (except for $\alpha = 1$, in case 2), increasing alpha reduces $\pi(\alpha)$ and brings it *closer* to zero, which amounts to reducing the relative size of $P(\alpha) > 0$. For example, in case 2, $\pi(\alpha)$ falls from $\pi(0) = 0.36$, to $\pi(0.5) = 0.22$, to $\pi(1) =$

[23] For a proof, see section A.6 in Appendix.
[24] For a proof, see section A.7 in Appendix.
[25] Precisely, $H = 0$ in cases 2, 3, and 4, as in each case the tax is α-neutral for an $\alpha \in [0,1]$ (see above where we stated that an α-neutral tax cannot cause reranking). And $H \approx 0$ in cases 1 and 5.

0. This means that according to the absolute inequality view, the progressivity of the tax offsets 36 percent of $N(0)$, the welfare loss that would obtain if the tax were absolutely neutral (i.e., 0-neutral or uniform). The share of the welfare loss offset by progressivity is smaller, 22 percent, for the intermediate view located halfway between the polar views, corresponding to $\alpha = 0.5$. And for the relative view, as the tax is relatively neutral (proportional) ($P(1) = 0$), the welfare loss that would obtain if the tax were relatively neutral, namely $N(1)$, is not offset at all ($\pi(1) = 0$).

On the contrary, in cases 4 and 5, the taxes are regressive for all α (except for $\alpha = 0$, in case 4). Here, increasing α decreases $\pi(\alpha)$ and bring it *farther* from zero, which amounts to increasing the relative size of $P(\alpha) < 0$. In case 4, for example, according to the absolute view, the tax is neutral, and thus does has no impact on welfare ($\pi(0) = 0$). Yet, according to the intermediate view corresponding to $\alpha = 0.5$, the tax is regressive, and the regressivity contributes to the welfare loss on top of $N(0.5)$, with the contribution amounting to 22 percent of $N(0.5)$: $\pi(0.5) = -0.22$. The contribution of regressivity to the welfare loss rises further by getting closer to the relative view, where the contribution amounts to 55 percent of the loss $N(1)$: $\pi(1) = -0.55$.

Figure 1: $\pi(\alpha)$ across α for five types of taxes



Notes: $\pi(\alpha)$ is defined in section 3.2. The five cases correspond to those discussed in section 3.1. D_t – concentration index of tax; G_x – Gini coefficient of pre-tax income.
Source: Authors' elaboration.

Case 3 is a mixed case: the tax is progressive for the values of α up to 0.34, for which it is neutral, and regressive for all values of α above this value. Thus, up to $\alpha = 0.34$, a higher α is associated with a smaller fraction of the loss $N(\alpha)$ being offset by the welfare-increasing impact of progressivity, as the relative size of $P(\alpha) > 0$ falls. Beyond $\alpha = 0.34$, raising α increases the relative size of $P(\alpha) < 0$, increasing thus the welfare-reducing impact of regressivity.

Equation (19) can be interpreted in another way as well. Let us call $\tau \coloneqq \mu_t/\mu_x$ the "overall ATR".[26] Now let us introduce what may be termed the "welfare equivalent overall ATR", denoted $\tau^*(\alpha)$. It is defined as the overall ATR associated with the α-neutral tax that would cause the same welfare loss, $W_y - W_x$, as the overall ATR associated with the actual tax ($\tau = \mu_t/\mu_x$). Formally, $\tau^*(\alpha)$ satisfies[27]

$$\frac{\tau}{\tau^*(\alpha)} = \frac{1}{-\delta(\alpha)} = \frac{1}{1 - \pi(\alpha) - \eta(\alpha)}. \tag{20}$$

According to (20), for a given welfare loss, the more (less) α-progressive the actual tax is, and the less (more) reranking it causes, the higher (lower) the actual overall ATR may be relative to the overall ATR associated with the α-neutral tax. In other words, for a fixed loss of social welfare that the government is willing to impose on the society, it can tax away a higher (lower) fraction of pre-tax income the more (less) progressive the tax is and the less (more) reranking takes place. Since $\tau^*(\alpha)$ depends on α, the ratio $\tau/\tau^*(\alpha)$ does, too. As noted above, both $\pi(\alpha)$ and $\eta(\alpha)$ are decreasing in α, so that $\tau/\tau^*(\alpha)$ is decreasing in α. Hence, for a given welfare loss, as we are increasing α the fraction of aggregate pre-tax income that is actually taxed away becomes smaller relative to the fraction that would be taxed away if the tax were neutral. Note that, since $\tau/\tau^*(\alpha) = \mu_t/\mu_t^*(\alpha)$, the previous sentence is equivalent to this one: For a given loss, as we are increasing α the actual average tax liability becomes smaller relative to the average tax liability that would prevail if the tax were neutral.

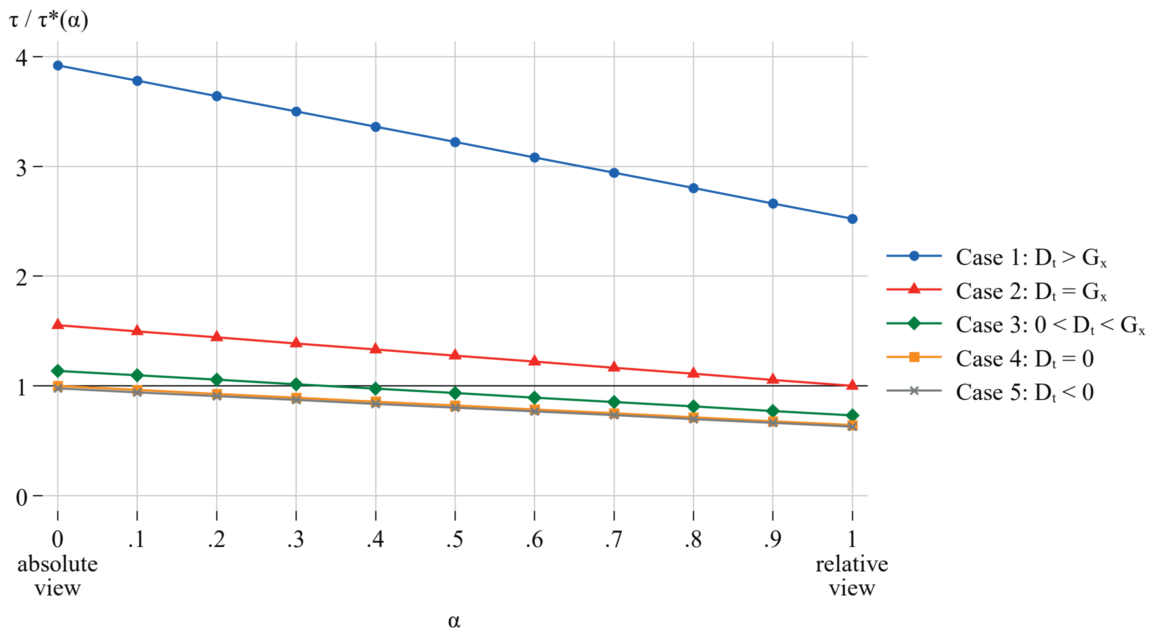
Figure 2 displays $\tau/\tau^*(\alpha) = \mu_t/\mu_t^*(\alpha)$ for the five cases considered in figure 1. Clearly, the more progressive the tax, the larger the $\tau/\tau^*(\alpha)$ ratio. Consider case 1, where the tax is more progressive than in the other cases for any inequality view. For the absolute view, τ is almost four times larger than $\tau^*(0)$, meaning that, if the progressive tax were replaced by the absolutely neutral tax that causes the same welfare loss, the average tax liability (and the total revenue from this tax) would have to be just (1/4)·100% = 25% of the actual. This percentage rises as we move from the absolute towards the relative view. For the relative view, if the actual tax were replaced by the relatively neutral tax equivalent to the actual in terms of the welfare loss, the implied average tax liability would amount to about (1/2.5)·100% = 40% of the actual figure. As another example, take case 4, where the tax is absolutely neutral, and regressive for all other

[26] Here "overall" signifies that that τ is the ATR pertaining to the entire distribution, rather the ATR specific to a particular level of pre-tax income, $\tau(x) = t(x)/x$.

[27] For a proof, see section A.8 in Appendix.

inequality views. Thus, except for the absolute view, where $\tau/\tau^*(0) = 1$, for all other views we have $\tau/\tau^*(\alpha) < 1$, meaning that for any inequality view but the absolute ($\alpha = 0$) one can, holding the welfare loss unchanged, replace the actual (regressive) tax with the neutral tax that collects more revenue.

Figure 2: $\tau/\tau^*(\alpha)$ across α for five types of taxes



Notes: $\tau/\tau^*(\alpha)$ is defined in section 3.2. The five cases correspond to those discussed in section 3.1. D_t – concentration index of tax; G_x – Gini coefficient of pre-tax income.
Source: Authors' elaboration.

At the beginning of section 3, we stated that all the analytical results hold equally for any admissible value of ρ, the single parameter of the S-Gini class of SWF. Here we just emphasise that, while the analytical results do not depend on ρ, the empirical results in concrete applications do in general depend on the parameter, as the values of the SWF's (W_x, W_y, Ψ_y, Ψ_t, Ω_α), and the Gini and concentration coefficients (G_x, G_y, D_y, D_t) are all ρ-specific. Thus, for instance, the inequality view for which a given tax is neutral, characterised by $\alpha = \tilde{\alpha} \coloneqq D_t/G_x$, depends on ρ since D_t and G_x do so. And similarly for other quantities appearing within the framework. So, for a ρ other than $\rho = 2$ that we have chosen, figures 1 and 2 would be quantitatively different. However, how much the figures would change for another ρ must be checked by concrete calculation, as the extent of change depends on the distribution of pre-tax income x and the tax function $t(x)$. For this reason, it is difficult to ascertain analytically how much the elements of equation (18a) would change upon a change in ρ. We conjecture that to derive analytical results, one would need to assume a parametric form for the pre-tax income distribution, a task going beyond the aim of this paper.

4 An application: tax progressivity and social welfare in Croatia

In this section, we apply the analytical framework developed in the preceding section to assess the welfare impact of taxation in Croatia in the year 2017.

4.1 The taxes, method, data, and definitions

We consider three taxes or, rather, three groups of taxes. The first group, which we refer to as "personal income tax" (PIT) includes the personal income tax and a related surtax. The Croatian personal income tax is levied on income from diverse sources, namely wage employment, self-employment, contractual work, rental and capital income, and pensions. In general, social benefits are not taxed. The tax base is gross income lowered by a basic personal allowance and a supplemental allowance for dependent family members. In the case of wage employment income, the amount of employee's social insurance contributions acts as a deduction. There is a general schedule and income source-specific schedules. The general schedule is used to tax, on a yearly basis, wage employment income, self-employment income, income from contractual work and pensions. This schedule consists of two bands: the lower band with a rate of 24 percent, and the higher band with a rate of 36 percent. For pensioners, the tax obligation amounts to 50 percent of the computed amount. The source-specific schedules are applied to rental and capital income, with a flat rate of 12 percent. The PIT obligation is the base for the surtax, which is paid at a rate set by the authorities at the level of towns and municipalities, with statutory restrictions on the maximum rate:[28] 10 percent for municipalities; 12 (15) percent for towns with a population below (above) 30 thousand.[29]

The second group of taxes, "social insurance contributions" (SIC), consists of the contributions for general health, occupational health, employment, and pension insurance. SIC are paid on income from employment, self-employment, and contractual work. In the case of self-employment income, SIC are paid as a lump sum, depending on type of self-employment, while for the other income types, the base is equal to gross income. The rates vary by type of SIC: 15 percent for the general health contribution; 0.5 percent for the occupational health contribution; 1.7 for employment contribution; and 20 percent for the pension insurance contributions.[30] Pensioners with a gross monthly pension exceeding the average national net monthly wage pay the special pensioner health contribution, whose rate is 3 percent.[31]

[28] There are no restrictions on the minimum rate. About half of all towns and municipalities have a zero surtax rate.
[29] The only exception is Zagreb, the capital, where the maximum allowed is 30 percent.
[30] Income from contractual work is subject to the general health and pension contributions only.
[31] For pensioners with gross monthly pension below the average national net monthly wage, the contribution rate is 1 percent, but the contribution is paid from the central government budget (CGB). CGB pays various contributions for other social groups as well, such as unemployed and persons on maternity/parental leave. However, neither of these CGB-financed contributions is included in our analysis.

The third group of taxes are indirect taxes (IND), namely the value added tax (VAT) and excises. The set of VAT rates includes the standard rate of 25 percent and two reduced rates of 5 and 13 percent. The 5 percent rate applies to basic food items such as bread and milk, medical drugs, and various merit goods such as books, newspapers, or cinema tickets. The rate of 13 percent applies to food items such as edible oils and fats, electricity, water supply, refuse collection, and services of the hospitality industry including bars, restaurants, and accommodation facilities. Services such as medical, educational, and financial, are exempt from VAT.[32] Excises are levied on electricity, oil products, alcoholic beverages, coffee and some other non-alcoholic beverages, and tobacco products.[33] In addition to the three groups of taxes, we also consider their combination (i.e., PIT + SIC + IND).

The taxes are simulated using EUROMOD, a tax-benefit microsimulation model for the European Union countries (Sutherland and Figari 2013).[34] Applying the rules of a country's system of taxes and social benefits to the EU-SILC[35] household survey data, the model simulates the amounts of taxes and social benefits that each household pays and receives, respectively. Here we use the Croatian component of EUROMOD (Urban, Bezeredi, and Pezer 2020).

As yet, the Croatian component of EUROMOD does not include the tool that allows for simulation of indirect taxes. To be able to simulate indirect taxes, we extend the default model by adding an indirect tax module, which is built following the Indirect Tax Tool version 2 (De Agostini et al. 2017).[36] To simulate indirect taxes we need for each household the information on consumption expenditures, and since the EU-SILC does not collect this information, it must be imputed from another survey. For that purpose, we use the Croatian Household Budget Survey for 2017, and impute consumption from it into the EU-SILC, following a parametric survey-to-survey imputation method set forth by De Agostini et al. (2017).[37]

To address the common issue of survey data not representing well incomes at the top of the distribution[38], the EU-SILC original incomes are corrected using a recent survey calibration method (Blanchet et al. 2019). The method makes the survey income distribution in line with the income distribution recorded in administrative data of the tax authority, which is deemed fully representative at the top. This is done

[32] We assume that pre-VAT prices are not affected by VAT, and thus treat the exempt services as zero-rated. Consequently, the possible cascading effects of the VAT paid on inputs in the production of the exempt services are not captured.
[33] Only the direct effects of excises are taken into account. For example, in the case of oil products or electricity, it is assumed that individuals pay the respective excises only through their direct consumption of oil products (as car fuel, for example) and electricity (in the household), but not indirectly through the consumption of goods and services produced using oil products or electricity as inputs.
[34] See also the EUROMOD website: https://euromod-web.jrc.ec.europa.eu/overview/what-is-euromod.
[35] European Union Statistics on Income and Living Conditions.
[36] Currently, there are two versions of the Indirect Tax Tool: ITT version 2 (De Agostini et al. 2017) and ITT version 3 (Acoğuz et al. 2020).
[37] See also Decoster, Ochmann, and Spiritus (2013; 2014).
[38] For a review, see Lustig (2019).

through a two-step procedure. In the first step, a data-driven algorithm determines the income level beyond which the survey income distribution is unrepresentative of the true distribution due to undercoverage of top incomes. The second step includes amending the survey weights attached to each interviewed household: households with top income earners are thus reweighted upward, while the rest are reweighted downward. At the same time, the survey remains representative of the underlying population in terms of demographic variables such as gender and age.[39] This enables us to capture more tax revenue, which would otherwise not be captured due to undercoverage of individuals with top incomes in the original survey data. Most notably, with the corrected data, we are able to simulate about 10 percent more revenue from personal income tax, getting thus closer to the total revenue actually raised.[40]

Before turning to the results, let us define the empirical counterparts of the theoretical concepts referred to in sections 2 and 3. Pre-tax income is the sum of gross (i.e., pre-PIT-and-SIC) income from all sources (wage employment, self-employment, financial assets, real property) and all social benefits received. Post-tax income depends on which tax is considered; thus, we have post-PIT income (pre-tax income minus PIT), post-SIC income (pre-tax income minus SIC), post-IND income (pre-tax income minus IND), and post-PIT-and-SIC-and-IND income (pre-tax income minus PIT minus SIC minus IND). The unit of analysis is an individual, assuming equal sharing among household members. All incomes and taxes are aggregated at the household level and then normalised by the OECD-modified[41] equivalence scale. So, whenever individuals are sorted by income (either pre- or post-tax), they are sorted by the equivalised household income.

4.2 The welfare impact of tax progressivity

We first describe the taxes in terms of how the ATR and the tax liability vary with pre-tax income. Figure 3 displays the amount of tax on panel A, and the ATR on panel B. Personal income tax (PIT) and social insurance contributions (SIC) are similar in the sense that for both the ATR and the tax liability tend to

[39] See Blanchet et al. (2019) for more details. Alternatively, a three-step procedure can also be applied, wherein the first two steps are as described above. In the third step certain number of synthetic observations, with incomes higher than the maximum income observed in the survey, are added to the survey, and the weights (the new ones from the second step) are uniformly downscaled so that the weights add up to the population. This, last step, can be interpreted as artificial oversampling of households with high-income individuals to overcome biases arising from insufficient sample size.
[40] More precisely, in the case of PIT from the sources subject to the general schedule (see above), we are able to close virtually the entire shortfall of the simulated aggregate amount from the amount recorded in administrative fiscal statistics as the actual collection. Considering the whole PIT (i.e., from all sources), a 10-percent shortfall still remains, however. This is due entirely to the fact that, for lack of information, we were not able to properly address the issue of undercoverage of top capital incomes, which are subject to a specific schedule (see above). Namely, while tax authority produces tabulations on distributions of employment, self-employment and pension income (which were all used in calibration), similar tabulations on capital income are not available.
[41] Number of adult equivalents according to the OECD-modified scale = 1 + 0.5×(number of adults – 1) + 0.3×number of children. Adults are persons aged 14 or more.

increase with pre-tax income. Thus, both are examples of case 1 in section 3.1. This is confirmed by the values of their concentration coefficients in table 1, which are both larger than the Gini coefficient of pre-tax income, and therefore both taxes are progressive for all inequality views in the continuum from the absolute to the relative.

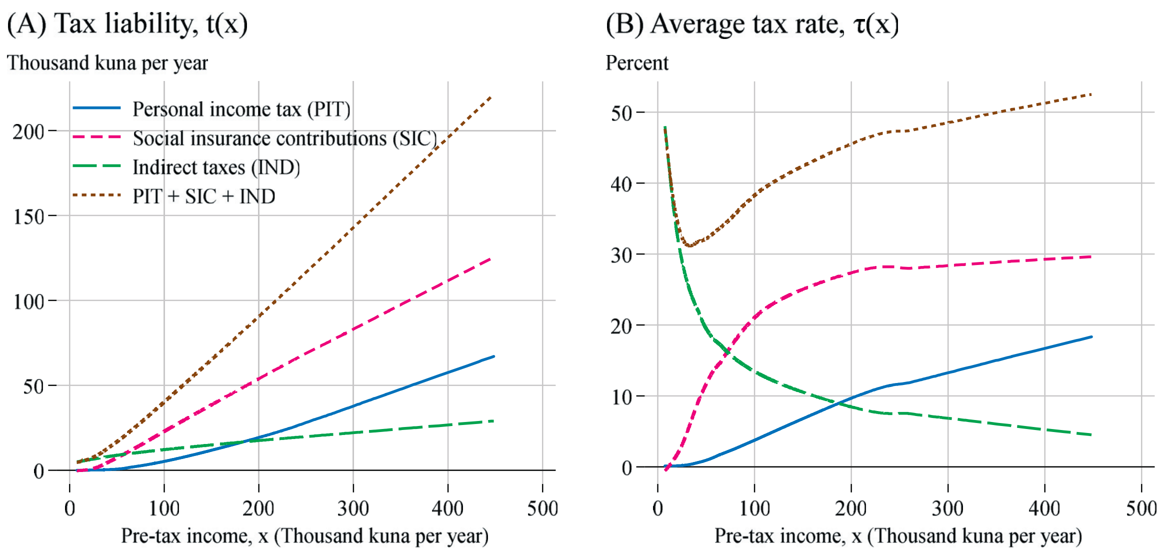
Unlike for PIT and SIC, in the case of indirect taxes (IND) the ATR decreases with pre-tax income, whereas the amount of tax, just like in the cases of PIT and SIC, increases with pre-tax income. Therefore, IND is an example of case 3, with a concentration coefficient which is positive and smaller than the Gini coefficient of pre-tax income, implying progressivity for all inequality views from the absolute to the one characterised by $\alpha_0 = 0.489$ (for which IND are neutral), and regressivity for all inequality views beyond it, all the way to the relative view.

When we combine PIT, SIC, and IND together, the tax-liability curve and the ATR curve are each obtained by adding up (vertically) the corresponding curves pertaining to PIT, SIC, and IND. While the resulting tax-liability curve is monotonically increasing as the other three, the ATR curve for the combined tax is J-shaped, where the downward-sloping part at the bottom of the pre-tax income distribution is entirely due to the poorest paying only IND, as they have non-zero consumption, but no income subject to PIT and SIC. Moving away from the very bottom, the ATR starts increasing, as PIT and SIC outweigh IND. Consequently, the concentration coefficient of the combined tax exceeds the Gini coefficient of pre-tax income, implying that the combined tax is progressive for any inequality view in the continuum from the absolute to the relative view.

In figure 4, we apply (18a)–(18e) to decompose the total welfare impacts of PIT, SIC, IND, and the combined tax (PIT + SIC + IND) into the α-neutral-tax effect, the α-progressivity effect, and the horizontal effect.[42] All the results are for the S-Gini parameter $\rho = 2$. Note first that the α-neutral-tax effect, $N(\alpha)$, is the same across the three taxes for a given inequality view. This is because, for a given α, it depends on the pre-tax income inequality, G_x, only. In addition, note that the taxes cause only a little reranking. Precisely, the horizontal effect, H, as a share of the welfare loss per one kuna of tax revenue, ΔW, equals 2.4, 3.2, 0.2, and 2 percent in the case of PIT, SIC, IND, and PIT + SIC + IND, respectively. Therefore, ΔW across the taxes and inequality views will be determined prevalently by the α-progressivity effect, $P(\alpha)$.

[42] The values on which the figure is based are given in table A1 in section A.9 in Appendix.

Figure 3: The average tax rate and the amount of tax across the distribution of pre-tax income



Notes: For descriptions of the taxes, see section 4.1. Each curve is obtained by smoothing the values on the ordinate using the LOWESS smoother with the default bandwidth size of 0.8. To prevent influential observations from the top and the bottom of the pre-tax income distribution from affecting the shape of the curves, the top and bottom 0.1 percent of observations were excluded before the smoothing. Pre-tax income and the tax liability are equivalized using the OECD-modified equivalence scale.
Source: Authors' estimates based on the tax-benefit microsimulation model EUROMOD and the EU-SILC 2017 data.

Table 1: Concentration of personal income tax, social insurance contributions, and indirect taxes in Croatia, 2017

Tax	Concentration coefficient of tax	Gini coefficient of pre-tax income
Personal income tax (PIT)	0.752	0.356
Social insurance contributions (SIC)	0.468	0.356
Indirect taxes (IND)	0.174	0.356
PIT + SIC + IND	0.405	0.356

Notes: For descriptions of the taxes, see section 4.1. The taxes and pre-fiscal income are equivalized using the OECD-modified equivalent scale.
Source: Authors' estimates based on the tax-benefit microsimulation model EUROMOD and the EU-SILC 2017 data.

Of the three taxes, PIT causes the smallest welfare loss per unit of tax collected, 0.25 kuna (i.e., $\Delta W = -0.25$) (panel A). This means that each kuna of the PIT revenue, when spent, must yield a return of 0.25 kuna – a 25-percent return – for the government to just break even in terms of social welfare. More than twice higher return of 0.55 kuna is required per unit of the SIC revenue for the government to break even (panel B). This is due to lower progressivity of SIC compared to PIT, for all inequality views, as measured by $P(\alpha)$, which is equal to the generalised Kakwani index of Urban (2019). When it comes to IND, the required break-even return is even higher, 0.83 kuna per one kuna of the IND revenue, as it is less progressive than both PIT and SIC for $\alpha < 0.5$, it is neutral for $\alpha = 0.5$, and regressive for all $\alpha > 0$ (panel C). Unlike progressivity, regressivity increases the welfare loss per unit of IND above $N(\alpha)$, the loss that would obtain if IND were neutral. When PIT, SIC, and IND are considered together, the combined tax is progressive for all α (panel D), coming close to neutrality for the relative inequality view. The decomposition is similar to that for SIC, with a bit smaller positive $P(\alpha)$ for all α, and hence a bit larger welfare loss per unit of tax revenue: 0.61 kuna (i.e., $\Delta W = -0.61$).

In figure 5 we plot $\pi(\alpha)$ against α for the four taxes.[43] Again, the results are for $\rho = 2$. PIT and SIC are both progressive for any inequality view, with PIT being considerably more progressive. The welfare-increasing impact of PIT's progressivity, as measured by $P(\alpha)$, offsets no less than 60 percent of the welfare loss that would obtain if PIT were neutral: for the absolute view, $P(0)$ offsets three quarters of the loss $N(0)$ ($\pi(0) = 0.751$), a bit short of 70 percent for $\alpha = 0.5$ ($\pi(0.5) = 0.697$), and a little more than 60 percent for the relative view ($\alpha(1) = 0.614$). Substantially smaller are the corresponding figures for SIC, namely $\pi(0) = 0.467$, $\pi(0.5) = 0.351$, and $\pi(1) = 0.172$, due to lower progressivity of SIC for any inequality view.

Unlike PIT and SIC, IND is progressive for nearly the half of inequality views that are closer to the absolute view ($\alpha < 0.489$), neutral for $\alpha = 0.489$, and regressive for about the half of inequality views closer to the relative view ($\alpha > 0.489$). Where progressive, IND is much less progressive than PIT and SIC, with at most 17 percent of the neutral-tax effect being offset by the progressivity effect ($\pi(0) = 0.174$). And where regressive, the contribution of IND to the welfare loss on top of the welfare-reducing neutral-tax effect reaches up to 28 percent of the neutral-tax effect ($\pi(1) = -0.283$).

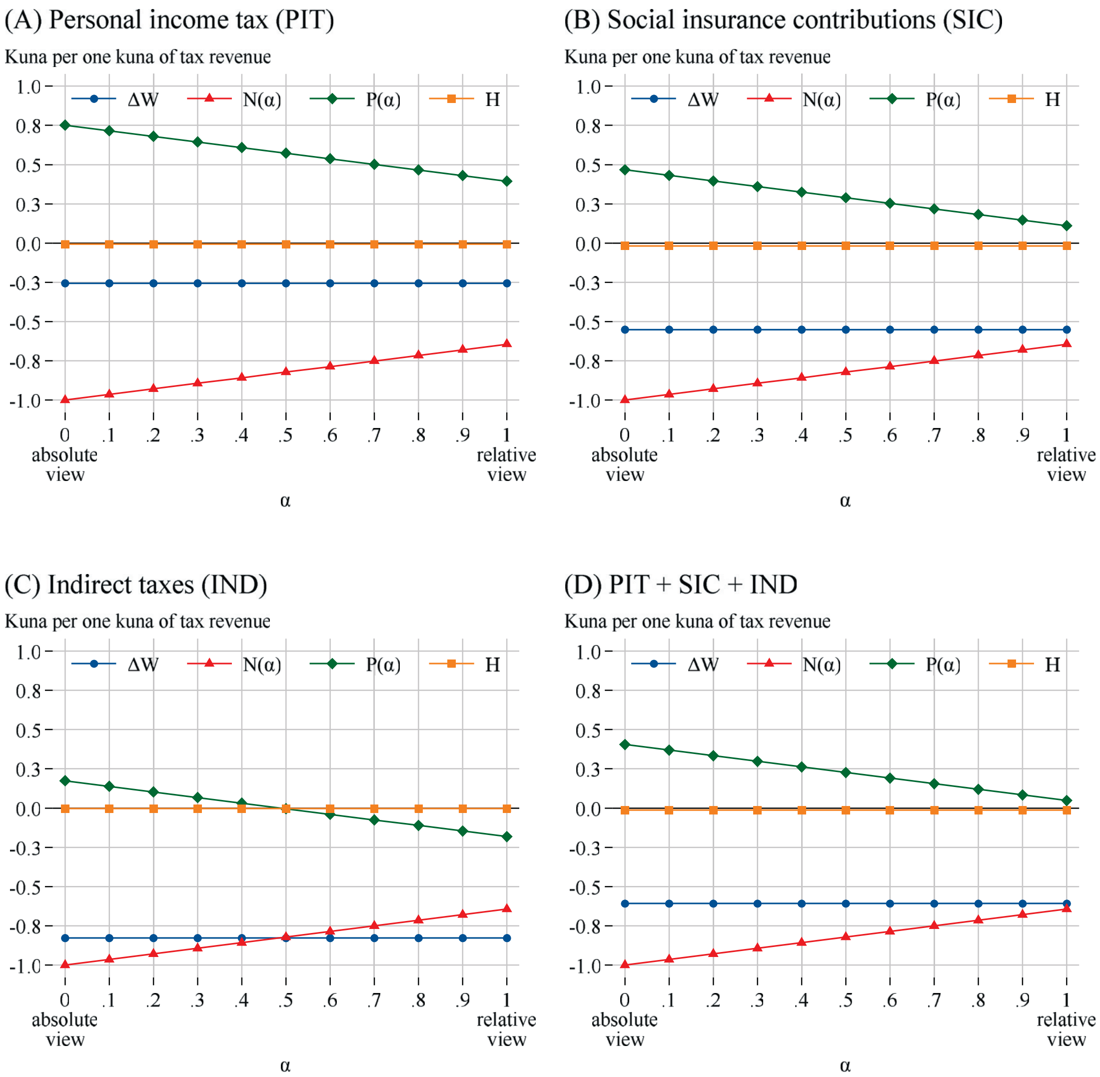
Despite this regressivity of IND for a subset of inequality views, the tax combining PIT, SIC, and IND into one (PIT + SIC + IND) remains progressive over the whole continuum of inequality views, just like PIT and SIC, but less than the latter two. Its progressivity is of a magnitude enough to offset 40 percent of the neutral-tax effect for the absolute view ($\pi(0) = 0.405$), about 28 percent for the intermediate view associated with $\alpha = 0.5$, while for the relative view the figure falls to about 7 percent ($\pi(1) = 0.075$).

Figure 6 displays the ratio $\tau/\tau^*(\alpha)$.[44] PIT, being the most progressive, exhibits substantially higher ratio than the other taxes. For the absolute view, PIT's progressivity allows the actual revenue to be 3.9 times higher than if the tax were absolutely neutral (i.e., uniform), while causing the same welfare loss. The ratio falls to 3.2 for the intermediate view $\alpha = 0.5$ and to 2.5 for the relative view. For SIC, progressive for all inequality views too, the progressivity also allows the actual revenue to be larger than in the case of welfare-loss-equivalent neutrality: about 80, 50, and 17 percent for $\alpha = 0$, $\alpha = 0.5$, and $\alpha = 1$, respectively (i.e., $\tau/\tau^*(0) = 1.82$, $\tau/\tau^*(0.5) = 1.49$, $\tau/\tau^*(1) = 1.17$). In the case of IND, its progressivity for the absolute view

[43] The values on which the figure is based are given in table A1 in section A.9 in Appendix.
[44] The values on which the figure is based are given in table A1 in section A.9 in Appendix.

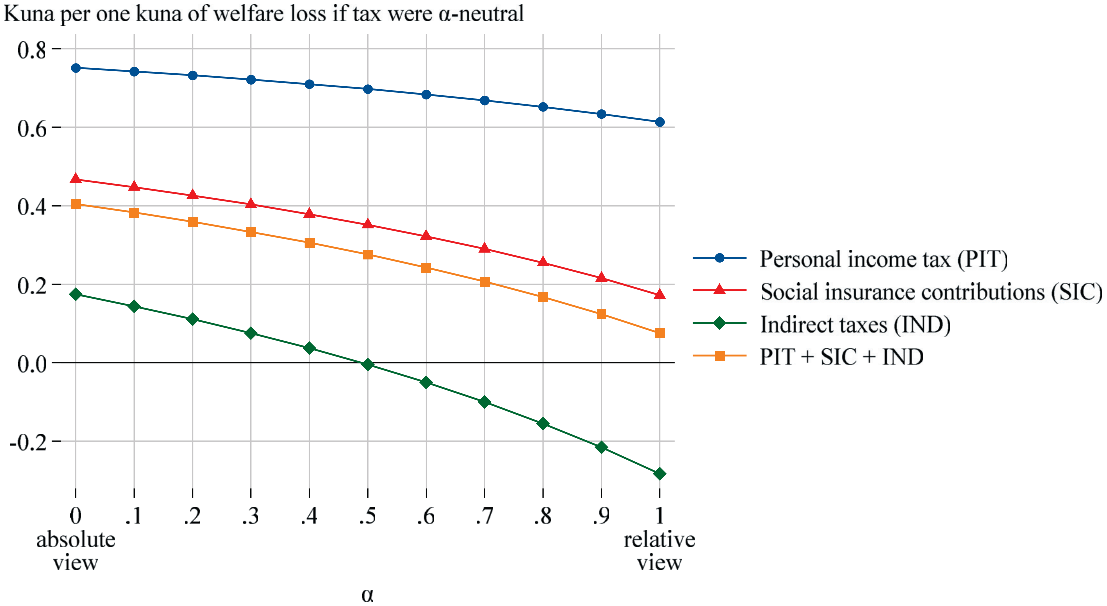
is such that the actual revenue exceeds by about 20 percent the revenue that would obtain if IND were neutral and caused the same welfare loss. This percentage falls practically to zero for the intermediate view $\alpha = 0.5$, while for the relative view, for which IND is regressive, the actual revenue falls 22 percent short of the revenue that would be possible to collect with the relatively neutral tax causing equal welfare loss. Finally, for the combined tax, PIT + SIC + IND, the ratio is, due to progressivity, above 1 for all inequality views, but comes close to 1 for the relative view: $\tau/\tau^*(0) = 1.65$, $\tau/\tau^*(0.5) = 1.35$, $\tau/\tau^*(1) = 1.06$.

Figure 4: Decomposition of the welfare impact of personal income tax, social insurance contributions, and indirect taxes



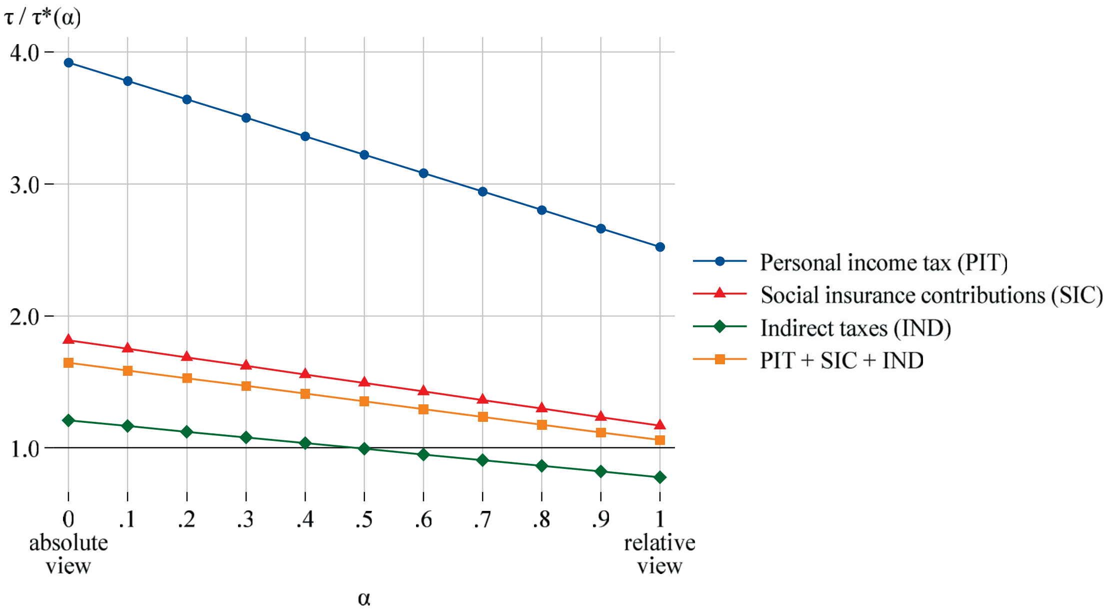
Notes: For descriptions of the taxes, see section 4.1. The decomposition is according to (18a–e).
Source: Authors' estimates based on the tax-benefit microsimulation model EUROMOD and the EU-SILC 2017 data.

Figure 5: $\pi(\alpha)$ across α for personal income tax, social insurance contributions, and indirect taxes in Croatia, in 2017



Notes: For descriptions of the taxes, see section 4.1. $\pi(\alpha)$ is defined in section 3.2.
Source: Authors' estimates based on the tax-benefit microsimulation model EUROMOD and the EU-SILC 2017 data.

Figure 6: $\tau/\tau^*(\alpha)$ across α for personal income tax, social insurance contributions, and indirect taxes in Croatia, in 2017



Notes: $\tau/\tau^*(\alpha)$ is defined in section 3.2. For descriptions of the taxes, see section 4.1.
Source: Authors' estimates based on the tax-benefit microsimulation model EUROMOD and the EU-SILC 2017 data.

In sum, the results indicate that the Croatian tax system is overall progressive. While PIT and SIC are progressive for all inequality views – with PIT being considerably more progressive – IND is progressive for about half of inequality views, namely those from the absolute view to approximately the intermediate view associated with $\alpha = 0.5$. If the Croatian tax system were neutral instead, and collected the same revenue, a larger welfare loss associated with taxation would have to be incurred. Also, replacing the actual, overall progressive tax system by the neutral system, subject to the constraint that the welfare loss remains the same as with the actual taxes, the revenue would have to be lower. Most importantly, the exact magnitudes depend on the inequality view taken, with notable differences not only between the figures pertaining to the relative and absolute views as the polar views, but also between the figures for more similar views.

5 Conclusion

If we accept that no feature of a public policy should be promoted as an end in itself, but rather as a means to increase a measure of social welfare, features of tax policy design should also be assessed in terms of the impact of taxes on social welfare. Thus, tax progressivity/regressivity, as a feature of tax design, should accordingly be evaluated considering its social welfare impact. This, however, has not traditionally been the case, as the standard approach to tax progressivity measurement, based on the Kakwani index of progressivity (Kakwani 1977), was not explicitly embedded in a social welfare framework and interpreted accordingly.

In a recent contribution, Kakwani and Son (2021) have developed a framework allowing for the decomposition of the social welfare loss due to taxation into a few constitutive elements, one of which pertains to tax progressivity/regressivity. Recognising that there are different views of inequality, they have provided the framework in two versions: one based on the relative inequality view, the other one on the absolute view.

In this paper, we have gone a step further, recognising that besides the absolute and relative inequality views as the polar cases there are many intermediate inequality views, namely a whole continuum ranging from the absolute to the relative view. We have generalised the Kakwani and Son (2021) framework by providing it in a form which nests the absolute and relative inequality views, as well as the whole continuum of intermediate views between them, as special cases. In the generalisation, we have made use of Urban's (2019) generalisation of the Kakwani index accommodating intermediate inequality views, where the underlying approach to intermediate inequality is based on the concept of α-inequality (Bosmans et al. 2014; del Río and Ruiz-Castillo 2000, 2001).

We have shown that the generalised decomposition of tax-induced welfare change can be expressed as a weighted average of the relative and absolute decompositions, derived by Kakwani and Son (2021), with the respective weights equal to α and $1-\alpha$. While the size of the welfare loss does not depend on inequality view, we find that its composition does. The relative importance of tax progressivity/regressivity varies with inequality view. Precisely, for a progressive tax, increasing α, which amounts to getting closer to the relative inequality view, reduces the relative importance of progressivity; conversely, for a regressive tax, getting closer to the relative view increases the relative importance of regressivity. Thus, the perception of the composition of a given tax-induced welfare loss varies with the inequality view taken.

In an empirical application examining the main taxes in Croatia, we have shown that it matters substantially which inequality view is taken in assessing the impact of taxation on social welfare. Overall, the Croatian tax system is progressive, and if it were neutral instead, the same amount of tax revenue would be possible to collect only at the cost of a larger welfare loss, the more so the closer the inequality view to the absolute view. Interpreted in a different way, if the actual, progressive tax system were replaced by the neutral system that causes the same welfare loss, the amount of tax revenue collected would have to be smaller, the more so the closer the inequality view to the absolute view. The progressivity of the overall tax system is due to the progressivity of personal income tax and social insurance contributions, which are progressive across all inequality views. Indirect taxes are progressive for approximately the half of inequality views closer to the absolute view, and regressive for the half of views closer to the relative view. However, the regressivity of indirect taxes for the subset of inequality views is not strong enough to render the overall tax system regressive for this subset of views.

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Appendix

A.1

Here we prove that $D_{t_\alpha} = \alpha G_x$.

By analogy with (7b) in section 2.1, D_{t_α} is given by

$$D_{t_\alpha} = 1 - \frac{\Psi_{t_\alpha}}{\mu_{t_\alpha}}, \tag{A1}$$

where

$$\begin{aligned}\Psi_{t_\alpha} &= \int_0^\infty t_\alpha(x)\omega(F(x),\rho)\, f(x)\mathrm{d}x = \int_0^\infty \left[\alpha\tau x + (1-\alpha)\tau\mu_x\right]\omega(F(x),\rho)\, f(x)\mathrm{d}x = \\ &= \alpha\tau \underbrace{\int_0^\infty x\omega(F(x),\rho)\, f(x)\mathrm{d}x}_{=W_x} + (1-\alpha)\tau\mu_x \underbrace{\int_0^\infty \omega(F(x),\rho)\, f(x)\mathrm{d}x}_{=1} \\ &= \alpha\tau W_x + (1-\alpha)\tau\mu_x.\end{aligned} \tag{A2}$$

Plugging (A2) into (A1) and using the fact that $\mu_{t_\alpha} = \mu_t$, we get

$$\begin{aligned}D_{t_\alpha} &= 1 - \frac{\alpha\tau W_x + (1-\alpha)\tau\mu_x}{\mu_{t_\alpha}} = 1 - \frac{\alpha\tau W_x + (1-\alpha)\tau\mu_x}{\mu_t} = 1 - \frac{\alpha\tau W_x + \tau\mu_x - \alpha\tau\mu_x}{\tau\mu_x} = \\ &= \frac{-\alpha\tau W_x + \alpha\tau\mu_x}{\tau\mu_x} = \alpha\left(1 - \frac{W_x}{\mu_x}\right) = \alpha G_x,\end{aligned} \tag{A3}$$

where the last equality is due to (6a) in section 2.1. ■

A.2

Here we prove that

$$\frac{1}{\mu_t}(\Omega_\alpha - W_x) = -(1 - \alpha G_x), \tag{A4}$$

$$\frac{1}{\mu_t}\left(\Psi_y - \Omega_\alpha\right) = D_t - \alpha G_x, \tag{A5}$$

$$\frac{1}{\mu_t}\left(W_y - \Psi_y\right) = \frac{\mu_y}{\mu_t}\left(D_y - G_y\right). \tag{A6}$$

We first show that (A4) holds:

$$\frac{1}{\mu_t}(\Omega_\alpha - W_x) = \frac{1}{\mu_t}\left(\int_0^\infty y_\alpha(x)\omega(F(x),\rho)f(x)\mathrm{d}x - \int_0^\infty x\omega(F(x),\rho)f(x)\mathrm{d}x\right) =$$

$$= \frac{1}{\mu_t}\left(\int_0^\infty \left[x - t_\alpha(x)\right]\omega(F(x),\rho)f(x)\mathrm{d}x - \int_0^\infty x\omega(F(x),\rho)f(x)\mathrm{d}x\right) =$$

$$= -\frac{1}{\mu_t}\int_0^\infty t_\alpha(x)\omega(F(x),\rho)f(x)\mathrm{d}x = -\frac{1}{\mu_t}\Psi_{t\alpha} = -\frac{1}{\mu_t}\mu_{t_\alpha}\left(1 - D_{t_\alpha}\right) = -(1 - \alpha G_x), \tag{A7}$$

where the last two equalities are due to equations (A1) and (A3), respectively. Next, we show that (A5) holds:

$$\frac{1}{\mu_t}(\Psi_y - \Omega_\alpha) = \frac{1}{\mu_t}\left(\int_0^\infty y(x)\omega(F(x),\rho)f(x)\mathrm{d}x - \int_0^\infty y_\alpha(x)\omega(F(x),\rho)f(x)\mathrm{d}x\right) =$$

$$= \frac{1}{\mu_t}\int_0^\infty \left[y(x) - \left(x - t_\alpha(x)\right)\right]\omega(F(x),\rho)f(x)\mathrm{d}x =$$

$$= \frac{1}{\mu_t}\left(-\int_0^\infty t(x)\omega(F(x),\rho)f(x)\mathrm{d}x + \int_0^\infty t_\alpha(x)\omega(F(x),\rho)f(x)\mathrm{d}x\right) =$$

$$= \frac{1}{\mu_t}\int_0^\infty \left[-t(x) + t_\alpha(x)\right]\omega(F(x),\rho)f(x)\mathrm{d}x = \frac{1}{\mu_t}\left(-\Psi_t + \Psi_{t_\alpha}\right) =$$

$$= \frac{1}{\mu_t}\left[-\mu_t(1 - D_t) + \mu_{t_\alpha}\left(1 - D_{t_\alpha}\right)\right] = D_t - D_{t_\alpha} = D_t - \alpha G_x \tag{A8}$$

where the last three equalities are due to equations (7b), (A1), (A3), and the fact that $\mu_{t\alpha} = \mu_t$. Finally, we show that (A6) holds:

$$\frac{1}{\mu_t}\left(W_y - \Psi_y\right) = \frac{1}{\mu_t}\left[\mu_y\left(1 - G_y\right) - \mu_y\left(1 - D_y\right)\right] = \frac{\mu_y}{\mu_t}\left(D_y - G_y\right), \tag{A9}$$

where the first equality is due to equations (6b) and (7a). ■

A.3

Here we prove that an α-neutral tax cannot cause reranking.

Without loss of generality, consider any two individuals, called A and B, with the pre-tax incomes x_A and x_B such that $x_A \geq x_B$. Suppose an α-neutral tax is levied, with the overall average tax rate τ and the mean tax liability μ_t. Their respective post-tax incomes are:

$$y_i = x_i - \alpha\tau x_i - (1 - \alpha)\mu_t, \quad i = A, B. \tag{A10}$$

Now suppose that the tax reranks A and B so that $y_A < y_B$. We have:

$$\begin{gathered} x_A - \alpha\tau x_A - (1-\alpha)\mu_t < x_B - \alpha\tau x_B - (1-\alpha)\mu_t \\ x_A(1 - \alpha\tau) < x_B(1 - \alpha\tau) \\ x_A < x_B \end{gathered} \tag{A11}$$

which contradicts $x_A \geq x_B$. ■

A.4

Here we prove that $\Delta W \leq 0$.

Suppose the opposite holds:

$$\Delta W = N(\alpha) + P(\alpha) + H > 0 \tag{A12}$$

$$-(1 - \alpha G_x) + (D_t - \alpha G_x) + \frac{\mu_y}{\mu_t}\left(D_y - G_y\right) > 0$$

$$D_t - 1 + \frac{\mu_y}{\mu_t}\left(D_y - G_y\right) > 0. \tag{A13}$$

By definition of the concentration coefficient, $D_t \in [-1,1]$; thus, $D_t - 1 \leq 0$. K&S prove that $\Psi_y \geq W_y$, which implies $D_y \leq G_y$; thus, $\frac{\mu_y}{\mu_t}\left(D_y - G_y\right) \leq 0$. Therefore, (A12) cannot hold. ■

A.5

Here we prove that $\Delta W = 0$ only if $D_t = 1$ and $H = 0$.

Suppose that $\Delta W = 0$. If so, then the left-hand size of (A13) is equal to zero:

$$D_t - 1 + \frac{\mu_y}{\mu_t}\left(D_y - G_y\right) = 0, \tag{A14}$$

which is true only if $D_t = 1$ and $H \coloneqq \frac{\mu_y}{\mu_t}\left(D_y - G_y\right) = 0$. ■

A.6

Here we prove that $\partial\pi(\alpha)/\partial\alpha < 0$, except if $D_t = 1$.

$$\frac{\partial\pi(\alpha)}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left(\frac{P(\alpha)}{|N(\alpha)|}\right) = \frac{\frac{\partial P(\alpha)}{\partial\alpha}|N(\alpha)| - P(\alpha)\frac{\partial|N(\alpha)|}{\partial\alpha}}{|N(\alpha)|^2} = \frac{-G_x|N(\alpha)| - P(\alpha)\frac{N(\alpha)}{|N(\alpha)|}\frac{\partial N(\alpha)}{\partial\alpha}}{|N(\alpha)|^2} =$$

$$= \frac{-G_x|N(\alpha)| - P(\alpha)\frac{N(\alpha)}{|N(\alpha)|}G_x}{|N(\alpha)|^2} = \frac{-G_x|N(\alpha)| - P(\alpha)(-1)G_x}{|N(\alpha)|^2} = \frac{G_x(P(\alpha) - |N(\alpha)|)}{|N(\alpha)|^2} =$$

$$= \frac{G_x(D_t - \alpha G_x - |-(1 - \alpha G_x)|)}{|N(\alpha)|^2} = \frac{G_x\left(D_t - \alpha G_x - (1 - \alpha G_x)\right)}{|N(\alpha)|^2} = \frac{G_x(D_t - 1)}{|N(\alpha)|^2}. \tag{A15}$$

Assuming $G_x > 0$, the derivative cannot be positive, as that would require $D_t > 1$, which cannot be true because, by definition of the concentration coefficient, $D_t \in [-1,1]$. The fraction can be either negative, when $D_t < 1$, or zero, when $D_t = 1$. ■

A.7

Here we prove that $\partial\eta(\alpha)/\partial\alpha < 0$, except if $H = 0$.

$$\frac{\partial\eta(\alpha)}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left(\frac{H}{|N(\alpha)|}\right) = \frac{\frac{\partial H}{\partial\alpha}|N(\alpha)| - H\frac{\partial|N(\alpha)|}{\partial\alpha}}{|N(\alpha)|^2} = \frac{-H\frac{\partial(1-\alpha G_x)}{\partial\alpha}}{|N(\alpha)|^2} = \frac{-H(-1)G_x}{|N(\alpha)|^2} = \frac{HG_x}{|N(\alpha)|^2}. \quad \text{(A16)}$$

Assuming, $G_x > 0$, the derivative is negative if $H < 0$, and zero if $H = 0$. ■

A.8

Here we prove that $\tau/\tau^*(\alpha) = 1/\big(-\delta(\alpha)\big)$.

Multiplying equation (18a) by μ_t, we obtain

$$\mu_t\Delta W = \mu_t N(\alpha) + \mu_t P(\alpha) + \mu_t H \quad \text{(A17)}$$
$$W_y - W_x = -\mu_t(1-\alpha G_x) + \mu_t(D_t - \alpha G_x) + \mu_y\big(D_y - G_y\big).$$

The welfare loss $W_y - W_x$ can as well be brought about by an α-neutral tax. By definition of α-neutral tax, this tax must be such that the average tax liability, $\mu_t^*(\alpha)$, is implicitly given by

$$W_y - W_x = -\mu_t^*(\alpha)(1-\alpha G_x). \quad \text{(A18)}$$

Thus, combining (A17) and (A18),

$$-\mu_t^*(\alpha)(1-\alpha G_x) = -\mu_t(1-\alpha G_x) + \mu_t(D_t - \alpha G_x) + \mu_y\big(D_y - G_y\big). \quad \text{(A19)}$$

Dividing (A19) by $-\mu_x(1-\alpha G_x)$, we get:

$$\frac{\mu_t^*(\alpha)}{\mu_x} = \frac{\mu_t}{\mu_x} + \frac{\mu_t}{\mu_x}\frac{D_t - \alpha G_x}{-(1-\alpha G_x)} + \frac{\mu_y}{\mu_x}\frac{D_y - G_y}{-(1-\alpha G_x)} = \tau - \tau\frac{D_t - \alpha G_x}{1-\alpha G_x} - \tau\frac{\mu_y}{\mu_t}\frac{D_y - G_y}{1-\alpha G_x} =$$
$$= \tau - \tau\frac{P(\alpha)}{|N(\alpha)|} - \tau\frac{H}{|N(\alpha)|} = \tau\big(1 - \pi(\alpha) - \eta(\alpha)\big) = -\big(-1 + \pi(\alpha) + \eta(\alpha)\big)\tau = -\delta(\alpha)\tau. \quad \text{(A20)}$$

Denoting $\mu_t^*(\alpha)/\mu_x \equiv \tau^*(\alpha)$, a straightforward reorganisation of (A20) gives (20). ■

A.9

See table A1.

Table A1: Data for figures 4, 5, and 6

α	ΔW	$N(\alpha)$	$P(\alpha)$	H	$\delta(\alpha)$	$\pi(\alpha)$	$\eta(\alpha)$	τ	$\tau^*(\alpha)$	$\tau/\tau^*(\alpha)$
Personal income tax (PIT)										
0	–0.255	–1.000	0.751	–0.006	–0.255	0.751	–0.006	0.045	0.011	3.920
0.1	–0.255	–0.964	0.716	–0.006	–0.265	0.742	–0.007	0.045	0.012	3.780
0.2	–0.255	–0.929	0.680	–0.006	–0.275	0.732	–0.007	0.045	0.012	3.641
0.3	–0.255	–0.893	0.644	–0.006	–0.286	0.722	–0.007	0.045	0.013	3.501
0.4	–0.255	–0.857	0.609	–0.006	–0.298	0.710	–0.007	0.045	0.013	3.361
0.5	–0.255	–0.822	0.573	–0.006	–0.310	0.697	–0.008	0.045	0.014	3.222
0.6	–0.255	–0.786	0.537	–0.006	–0.324	0.684	–0.008	0.045	0.015	3.082
0.7	–0.255	–0.751	0.502	–0.006	–0.340	0.669	–0.009	0.045	0.015	2.942
0.8	–0.255	–0.715	0.466	–0.006	–0.357	0.652	–0.009	0.045	0.016	2.803
0.9	–0.255	–0.679	0.431	–0.006	–0.376	0.634	–0.009	0.045	0.017	2.663
1	–0.255	–0.644	0.395	–0.006	–0.396	0.614	–0.010	0.045	0.018	2.523
Social insurance contributions (SIC)										
0	–0.551	–1.000	0.467	–0.018	–0.551	0.467	–0.018	0.236	0.130	1.816
0.1	–0.551	–0.964	0.432	–0.018	–0.571	0.447	–0.019	0.236	0.135	1.751
0.2	–0.551	–0.929	0.396	–0.018	–0.593	0.426	–0.019	0.236	0.140	1.686
0.3	–0.551	–0.893	0.360	–0.018	–0.617	0.403	–0.020	0.236	0.146	1.622
0.4	–0.551	–0.857	0.325	–0.018	–0.642	0.379	–0.021	0.236	0.152	1.557
0.5	–0.551	–0.822	0.289	–0.018	–0.670	0.352	–0.022	0.236	0.158	1.492
0.6	–0.551	–0.786	0.253	–0.018	–0.701	0.322	–0.023	0.236	0.165	1.427
0.7	–0.551	–0.751	0.218	–0.018	–0.734	0.290	–0.024	0.236	0.173	1.363
0.8	–0.551	–0.715	0.182	–0.018	–0.770	0.255	–0.025	0.236	0.182	1.298
0.9	–0.551	–0.679	0.146	–0.018	–0.811	0.216	–0.026	0.236	0.191	1.233
1	–0.551	–0.644	0.111	–0.018	–0.856	0.172	–0.028	0.236	0.202	1.169
Indirect taxes (IND)										
0	–0.827	–1.000	0.174	–0.002	–0.827	0.174	–0.002	0.131	0.109	1.209
0.1	–0.827	–0.964	0.139	–0.002	–0.858	0.144	–0.002	0.131	0.113	1.165
0.2	–0.827	–0.929	0.103	–0.002	–0.891	0.111	–0.002	0.131	0.117	1.122
0.3	–0.827	–0.893	0.067	–0.002	–0.926	0.076	–0.002	0.131	0.122	1.079
0.4	–0.827	–0.857	0.032	–0.002	–0.965	0.037	–0.002	0.131	0.127	1.036
0.5	–0.827	–0.822	–0.004	–0.002	–1.007	–0.005	–0.002	0.131	0.132	0.993
0.6	–0.827	–0.786	–0.039	–0.002	–1.052	–0.050	–0.002	0.131	0.138	0.950
0.7	–0.827	–0.751	–0.075	–0.002	–1.102	–0.100	–0.002	0.131	0.145	0.907
0.8	–0.827	–0.715	–0.111	–0.002	–1.157	–0.155	–0.003	0.131	0.152	0.864
0.9	–0.827	–0.679	–0.146	–0.002	–1.218	–0.215	–0.003	0.131	0.160	0.821
1	–0.827	–0.644	–0.182	–0.002	–1.286	–0.283	–0.003	0.131	0.169	0.778
PIT + SIC + IND										
0	–0.608	–1.000	0.405	–0.012	–0.608	0.405	–0.012	0.412	0.250	1.646
0.1	–0.608	–0.964	0.369	–0.012	–0.630	0.383	–0.013	0.412	0.260	1.587
0.2	–0.608	–0.929	0.333	–0.012	–0.654	0.359	–0.013	0.412	0.270	1.528
0.3	–0.608	–0.893	0.298	–0.012	–0.680	0.333	–0.014	0.412	0.280	1.470
0.4	–0.608	–0.857	0.262	–0.012	–0.709	0.306	–0.015	0.412	0.292	1.411
0.5	–0.608	–0.822	0.227	–0.012	–0.739	0.276	–0.015	0.412	0.305	1.352
0.6	–0.608	–0.786	0.191	–0.012	–0.773	0.243	–0.016	0.412	0.318	1.294
0.7	–0.608	–0.751	0.155	–0.012	–0.810	0.207	–0.017	0.412	0.334	1.235
0.8	–0.608	–0.715	0.120	–0.012	–0.850	0.167	–0.017	0.412	0.350	1.176
0.9	–0.608	–0.679	0.084	–0.012	–0.895	0.124	–0.018	0.412	0.369	1.118
1	–0.608	–0.644	0.048	–0.012	–0.944	0.075	–0.019	0.412	0.389	1.059

Notes: For definitions of the quantities in each column, see sections 2 and 3.

Source: Authors' estimates based on the tax-benefit microsimulation model EUROMOD and the EU-SILC 2017 data.

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